ASSIGNMENT –04
BASED ON BETA-GAMMA FUNCTIONS

1. Evaluate following using Beta-Gamma functions

a) $\overline{|4.5}$ b) $\left|\overline{\frac{-7}{2}}\right.$ c) $\int_0^\infty \sqrt[3]{x}\, e^{-\sqrt[6]{x}}\, dx$ d) $\int_0^1 \sqrt[3]{x \ln\left(\frac{1}{x}\right)}\, dx$ e) $\int_0^1 \left(1-x^n\right)^{\frac{1}{n}} dx$

2. Prove following:

a) $\int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{\sin\theta}} \times \int_0^{\frac{\pi}{2}} \sqrt{\sin\theta}\, d\theta = \pi$ b) $ma^{\left(\frac{n+1}{m}\right)} \int_0^\infty x^n e^{-ax^n} dx = \left|\overline{\frac{n+1}{m}}\right.$

BASED ON LAPLACE TRANSFORMS

1. Find Laplace Transform of $f(t) = \begin{cases} t & 0 \le t < 1 \\ 2-t & 1 \le t < 2 \\ 0 & 2 \le t \end{cases}$.

2. Find Laplace Transform of following :

a) $\sin 2t.\cos 4t$ b) $\sin^3 t$ c) $t^{\frac{3}{2}} + t^{\frac{1}{2}} + 2$ d) $e^{2t}\cos^2 t$

e) $e^{\frac{3}{2}t}\cos 5t \sin 3t$ f) $\sin 6t.\cosh 3t$ g) $e^{2t}(t^2+t+2)$. h) $t^3 \sinh 2t$

i) $t^{\frac{5}{2}} e^{3t}$ j) $te^{2t}\sin 2t\cos t$ k) $\int_0^t e^{-t}\sin 3t\, dt$ k) $\left(t^2-3t+2\right)\sin 3t$

l) $\frac{e^{2t}\sin 2t\cos t}{t}$ m) $\frac{1-e^t}{t}$ n) $\int_0^t te^{-4t}\sin 3t\, dt$ o) $e^{-4t}\int_0^t t\sin 3t\, dt$

p) $t\int_0^t e^{-4t}\sin 3t\, dt$

3. Find Inverse Laplace transform of following :

a) $\frac{s+7}{(s+3)(s+2)}$ b) $\frac{s+2}{s^2+4s+20}$ c) $\frac{s}{(s+1)^{3/2}}$

d) $\frac{s+3}{(s^2+2s+1)(s^2+2s-1)}$ e) $\frac{1}{s^4+s^2+1}$ f) $\frac{1}{2s^2} - \frac{(\sqrt{s}-1)^2}{s^5} + \frac{3}{s+1}$

g) $\frac{36}{s(s^2+1)(s^2+9)}$ h) $\frac{1}{s^2}\left(\frac{s+1}{s^2+1}\right)$ i) $\ln\left(\frac{s+a}{s+b}\right)$

j) $\int_s^\infty \ln\left(\frac{s+2}{s+1}\right) ds$ k) $\tan^{-1}\left(\frac{2}{s^2}\right)$ l) $\cos^{-1}\left(\frac{s+4}{5}\right)$

m) $\frac{5}{(s-2)^6}$ n) $\frac{s^2}{(s^2+a^2)(s^2+b^2)}$ o) $\frac{1}{(s^2+4)(s+1)^2}$

4. If Laplace transform of $f(t) = F(s)$ then prove $L\{\cosh at\, f(t)\} = \frac{1}{2}\{F(s-a)+F(s+a)\}$.

5. Evaluate following integrals using Laplace transform

a) $\int_0^\infty e^{-2t}\sin 2t\, \cos 4t\, dt$ b) $\int_0^\infty t\, e^{-3t}\cos t\, dt$ c) $\int_0^\infty \frac{e^{-3t}\cos t}{t} dt$

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